Computational and Physical Modeling of the Compass Gait Passive Dynamic Walker

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***Abstract*—A compass-gait walker is type of bipedal passive dynamic walker known for its simplicity. We created a 2D mathematical model of a compass gait walker using the "Simplest Walking Model" proposed by Garcia et. al. [1]. To supplement the computational model, we built a 3D compass gait walker using alterations delineated in "Actuating a Simple 3D Passive Dynamic Walker" by Tedrake et. al [3]. In both the physical and computational model we were able to achieve the expected motion. We find that although the computational model requires very precise conditions to function, stable representations of the physical model can be achieved with simple alterations.**

I. Background

Passive dynamic walkers comprise an entire area of research in the field of mechanical simulation and rigid-body dynamics. This project was inspired the desire of both authors to build something physical that could be mathematically modeled and explained in the terms of concepts learned in the past semester.

As its name suggests, the Simplest Walking Model by Garcia et. al. [1] is one of the simplest models of passive dynamic bipedal walking that exists. The motion is simply achieved using the compass gait walking model (further detailed in the paper "Limit cycles and their stability in a passive bipedal gait" [2]).

To build a real-world model of a compass-gait passive dynamic bipedal walker, we modeled one by taking cues from existing physical walkers that resembled the simplest walking model. These physical models stray from the simple model most notably in that they require motion in the frontal plane. This motion is important to the success of the walker but does not heavily affect the side-plane compass gait motion. Such motion of a dynamic walker in two planes is further detailed in "Actuating a Simple 3D Passive Dynamic Walker" (Fig. 2), which was heavily consulted in the design of our physical model [3].

Fig. 1: An example of a passive dynamic walker with knees and jointed feet. This model is much more complex than the one we chose to study.

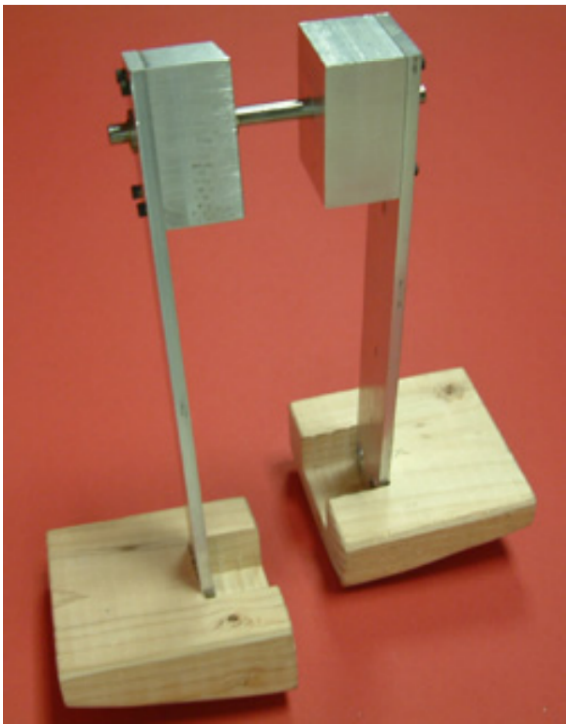

Fig. 2: The 3D Passive Dynamic Walker, designed by Tedrake et. al., was the main inspiration for the construction of our physical model.

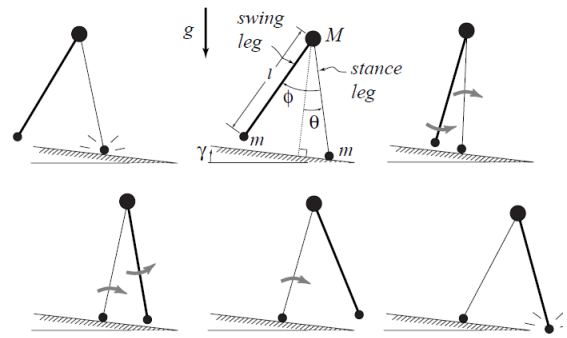

Fig. 3: An illustration from "The Simplest Walking Model: Stability Complexity, and Scaling," Garcia. et. al.: "A typical passive walking step. The new stance leg (lighter line) has just made contact with the ramp in the upper left picture. The swing leg (heavier line) swings until the next heelstrike (bottom right picture)." [1]

II. Learning Objectives

For this project, we had several broad learning goals, as well as a few goals specific to our topic of passive dynamic walkers. In a broad sense, we wanted to model the system in multiple ways. This includes computationally modeling the system using ODEs as well as creating a physical model of the system. Our goal was to compare these two models qualitatively in order to use each model to explain phenomena in the other. More specifically to passive dynamic walkers, our primary goal was to learn about bipedal walking motion, which, from an empirical standpoint, seems difficult.

III. System Models

A. The Compass Gait Walker

Of the many types of dynamic walking that have been studied, we examined the compass gait model. Figure 3 demonstrates the walking motion of this model. In this model, a mass (the "body" of the walker) is suspended in the air by two rigid legs. At any point in time, one of these legs is planted firmly on the ground and does not slide while the other is swinging through the air in the direction of walking.

At the beginning of each step (and the end of the step before it), both feet are instantaneously in contact with the ground. We refer to this point in time as the heelstrike. During this moment of each step, the legs switch roles; the swing angle becomes the previous stance angle and vice versa; the angular velocities of both angles change in order to conserve the angular and linear momentum of the walker.

B. Assumptions and Limitations

A few assumptions must be made for the sake of the models simplicity. This two-dimensional model assumes that the mass is located at the pivot and the axis of pivot is pointing into or out of the page. Additionally, during a step, the swinging leg of the walker is allowed to pass through the ground. Conveniently, an alteration of the design of the physical model allowed us to circumvent both of these limitations.

C. Equations of Motion

All equations come from Garcia, et al. [1]
Equations of motion, rescaled by $\sqrt{\frac{l}{g}}$:

$$\ddot{\theta} = sin(\theta) \tag{1}$$

$$\ddot{\phi} = \ddot{\theta} + \dot{\theta}^2 sin(\phi) - cos(\theta) sin(\phi) \tag{2}$$

Collision equation:

$$2\theta - \phi = 0 \tag{3}$$

Heelstrike equations, where a $^-$ indicates the value before collision, and a $^+$ indicates the value after collision:

$$\theta^+ = \theta^- - \phi^- \tag{4}$$
$$\dot{\theta}^+ = \dot{\theta}^- cos(2\theta^-) \tag{5}$$
$$\phi^+ = -\phi^- \tag{6}$$
$$\dot{\phi}^+ = \dot{\theta}^- cos(2\theta^-)(1 - cos(2\theta^-)) \tag{7}$$

D. Computational Simulation in MATLAB

We used ODE45 to numerically integrate the ODEs for each step, stopping the integration when the collision condition was reached. At this point, the heelstrike system of equations was performed on the end conditions before moving to the next step.

We employ a solver that, using *fsolve*, assesses the stability of the model by examining conditions after a single step, to find initial conditions that will yield stable walking for more than a few steps. Without this solver, finding initial conditions that result in stable walking would be difficult.

E. Physical Model

For passive walking in the physical world, we use a model detailed in Actuating a Simple 3D Passive Dynamic Walker [3], shown in Figure 2, which employs an oscillatory motion in the front plane to maintain the stability of the pivot axis and give each leg enough space to not collide with the floor. Additionally, with the assumption that the legs have enough space to swing, they are given curved feet (from the side plane point of view) for increased stability. The result of the necessary oscillation of the model from the front plane and curvature of the feet in the side plane was to construct the feet out of spherical sections. In our working model, these sections are from a sphere of radius 30 in. (0.76 m). Figure 5 shows the finished physical model.

F. Coordinate System and Nomenclature

We chose to use the coordinate system and naming conventions delineated in Figure 3. The stance leg is the leg that is planted in the ground while the other leg, the swing leg, moves through the air. The position of the system at any point in time can be given with two state variables: θ, referring to the angle between the stance leg and a line perpendicular to the ramp, and ϕ, referring to the angle between the stance leg and the swing leg.

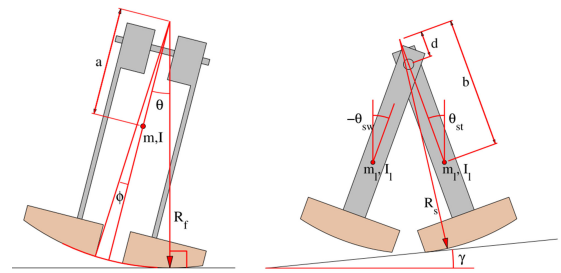

Fig. 4: Illustration of front plane view (left) and side plane view (right) of "Simple 3D Passive Dynamic Walker" (Tedrake et. al.) [3]. In the front plane, the curvature of the feet along this plane facilitates oscillatory rocking motion that prevents the swing foot from colliding with the floor. The feet are also curved along the side plane to provide stability to the walker.

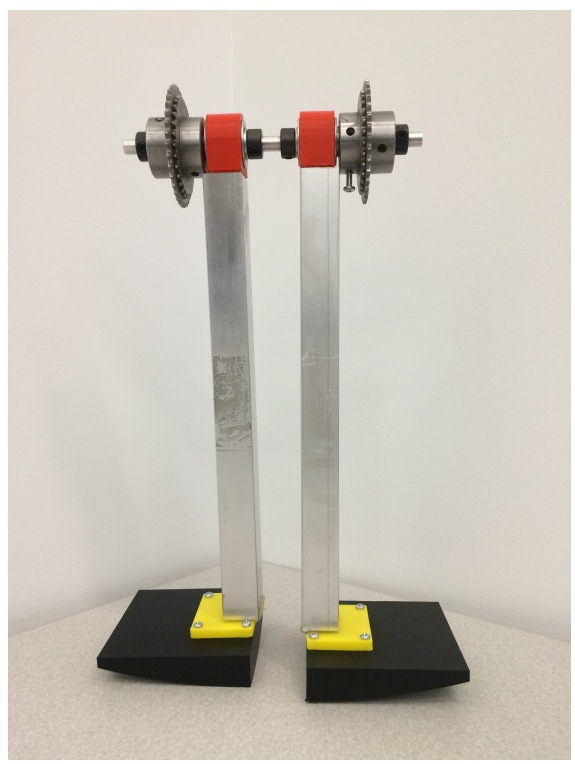

Fig. 5: The finished, functional bipedal compass gait walker.

IV. Results & Discussion

A. Computational Simulation

We were able to successfully simulate a compass gait model for many steps. These results can be seen in Figure 6. Each repeated section represents one step, where θ is the stance angle and ϕ is the swing angle. As expected because of our initial conditions, each step is the same as the previous, in order to achieve many steps.

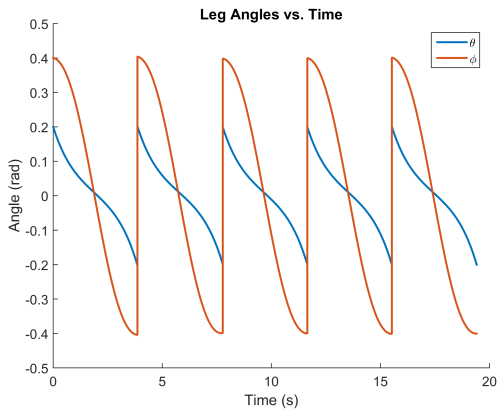

Fig. 6: The stance and swing leg angles for five steps of a compass-gait walker. The initial conditions have been tuned such that the initial condition for each step is the same. This ensures that the walker is able to take many steps without becoming unstable

The reason that the angles seem to jump between steps is that when the swing leg contacts the ground, the leg that was the stance leg is now called the swing leg, and the leg that was called the swing leg is now called the stance leg. When this change occurs, the angles θ and ϕ are also changed accordingly.

This model only worked for small values of γ. This seemed to be because the additional incline of the ramp adds energy into the system, resulting in a walker that gradually increases in velocity, and thus step size, and eventually falls. The value of γ used for the above figure was 0.009 rad (0.516^o).

B. Physical Model

While our model was initially unsuccessful at walking more than a single step, we made modifications that enabled it to walk 16 steps, a distance of over a meter from where it started.

A single modification to the feet, increasing the radius of the spherical sections from 15 inches to 30 inches, enabled it to walk independently for multiple steps. This change also made the walker able to stand by itself which it previously was unable to do.

A further improvement in the independent traveling distance of the walker came as result of adding mass (in the form of gear sprockets) to the pivot rod. Adding these masses to the outside faces of the legs (as shown in Fig. 5) proved to be most effective, as the masses lent an increased moment of inertia of the walker about the vertical axis, and thus more resistance to veering off course. Additionally, these masses shifted the center of mass of the walker closer to the pivot, making it more closely resemble the simplified model.

In testing the walker at various angles, we found that only small ramp angles resulted in stable motion. The walker performed best at an angle of about 0.0176 rad (1.01 degrees). For angles greater than this, the walker would lurch forward during its first few steps and its upper mass would pitch too far forward to be able to be supported. For smaller angles, the walker did not perform as well because it was not provided with a constant source of kinetic energy. Please refer to the videos in the course folder for a demonstration of the walker traveling down a slope.

C. Insights from Comparison of Computational and Physical Models

While we gleaned many individual insights from the computation and physical models, we learned just as much through the comparison of the two models. One such insight was in regard to the rotation of the physical model around its vertical axis when walking (detailed above). The computational model only shows 2D motion, and therefore does not provide any information about this movement. However, one large assumption that was made in the computational model was that the mass of the pivot is much larger than the masses of the feet. If this assumption were to be implemented on a physical model the outcome would be little to no rotation around the vertical axis of the walker. This also gives a reason to why the $\beta = 0$ model is appropriate.

V. Visualization

Please see course folder for an animation of the computational simulation and two recorded videos of the working physical model.

VI. Diagnosis

In the computational model, we found that the initial conditions must be very specific in order to simulate more than a few steps. While the model could achieve one step with a wide range of initial conditions, the heelstrike equations that dictate the initial conditions of the next step based on final conditions of the current step do not often yield conditions that enable an additional step. Even if an additional step is possible, the steps

can quickly degrade until the walker falls; for example, each stride could decrease in length. However, if the initial conditions of each step were to be the same, the walker would theoretically be able to take an infinite amount of steps. Following a code example from Cornell University [1], we created a short function to search for an initial condition that fulfilled this condition. This functions uses the fsolve function to find where the given initial condition equals the initial condition for the next step, given by the heelstrike equation.

VII. Improvement

The difference between the computer simulation and the physical model indicates the improvements that could be taken to improve the physics model that we employed. First, adding the frontal plane oscillatory motion into the simulation, and thus making the model three-dimensional, would allow for even more additions, such as giving feet to the model. These changes would lead to a more accurate–and possibly more stable–model.

VIII. Reflection

We learned a great deal about the process of taking a concept and turning it into both a computational and a real-life model. From the physics involved in recreating a passive dynamic walker, we learned that models can get complex very quickly; even changing the position of the mass or accounting for mass in both legs would make this problem much more complex, not to mention adding knee joints, foot joints, or arms. From constructing our own passive dynamic walker, we learned how to assess and improve a physical model; seeing that the walker was unstable and oscillating too aggressively in its initial iteration, we made the feet flatter and dramatically improved the performance of the walker.

IX. Conclusion

Within the timeframe of this project, we have been able to develop two representations of a compass-gait walker: a MATLAB simulation as well a physical model. Both of these models were able to successfully demonstrate the motion observed in previous studies. From these models, we found that while the simulation requires exact initial conditions, the physical model can work with a wider range of conditions. In addition, we were able to use each model to explain features of the other, linking our connection between the mathematical and physical worlds.

X. Future Usage

We believe that this project would be appropriate for students in a future Dynamics class. With the simplified model and the assistance of outside resources, it is possible to derive the equations of motion in order to create a mathematical model. In addition, the creation of the physical model is a manageable and enjoyable task that we would recommend to any group. We believe that in the future, students could go one step further to compare their computational and physical models quantitatively through the use of a program such as Tracker.

References

[1] M. Garcia, A. Chatterjee, A. Ruina, M. Coleman, The Simplest Walking Model: Stability Complexity, and Scaling, Cornell Theoretical and Applied Mechanics, 1998.

[2] A Goswami, B. Espiau, A. Keramane, Limit cycles and their stability in a passive bipedal gait, IEEE International Conference on Robotics and Automation, 1996, p. 246-251.

[3] R. Tedrake, T.W. Zhang, M. Fong, and H.S. Seung, Actuating a Simple 3D Passive Dynamic Walker, MIT CSAIL, 2004.